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EQUIVALENT CIRCUIT ANALYSIS OF A WT-2 PIEZOELECTRIC TRANSDUCER.(U)
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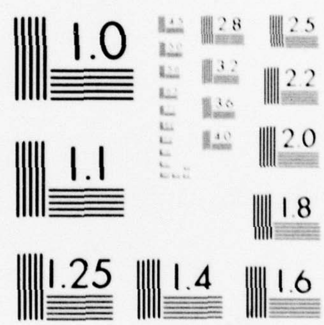

MICROCOPY RESOLUTION TEST CHART
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Hudson Laboratories
of
Columbia University
Dobbs Ferry, New York

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Alan Berman
Director

9 Technical Memorandum No. 69

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EQUIVALENT CIRCUIT ANALYSIS OF A WT-2 PIEZOELECTRIC TRANSDUCER

by

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UNCLASSIFIED

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INTRODUCTION

An equivalent circuit is invaluable in the design of a sound source and in the prediction of its behavior when subject to an acoustic load.

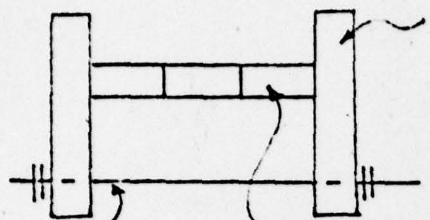
A piezoelectric transducer may actually be represented by means of two different equivalent circuits each of which will have the same impedance characteristics as the source.

The attached computations show in detail how to compute the parameters of the two equivalent circuits for the WT-2 (1200) piezoelectric transducer designed by P. Weber. The impedance of one of the two equivalent circuits is evaluated at various frequencies for the unloaded condition and the impedance loop thus obtained is compared with the source in-air measurements. Adjustments are made in the circuit to make its impedance match the actual source characteristics as closely as possible.

The circuit is then modified by the addition of the theoretical acoustic water load and the impedances of this circuit are compared with measurements on the source in water.

Good agreement between theory and measurements was obtained in the analysis of the WT-2 (1200) and WT-2 (400) transducers.

ANALYSIS OF WT-2 (1200)



MASS = 12.8# = 5.8 KG

3 - PZT.4 CRYSTALS IN PARALLEL
$\frac{1}{8}$" WALL $1\frac{1}{2}$" OD , $2\frac{1}{4}$" LONG

$K_{free} = 1300$ $k_{31} = .33$

$Y_{sc} = 8.2 \cdot 10^{10} \frac{NEWT}{M^2}$

BIAS ROD:
PHOSPHOR BRONZE
$\frac{3}{8}$" ϕ , $8\frac{3}{4}$" LONG

$Y_R = 15 \cdot 10^6 psi = 10.3 \cdot 10^{10} \frac{NEWT}{M^2}$

PARAMETERS FOR EQUIVALENT CIRCUITS:

$$C_E = n K_{free} \epsilon \frac{A}{t} = 3 \cdot 1300 \cdot 8.85 \cdot 10^{-12} \frac{\pi (1.25)\, 2.25}{\frac{1}{8} \quad 39.4} = .0745 \mu F$$

$$C_M' = \frac{l}{A Y_{sc}} = \frac{6.75 \cdot 39.4 \cdot 4}{\pi (1.50^2 - 1.25^2)\, 8.2 \cdot 10^{10}} = .0060 \cdot 10^{-6} \frac{M}{NEWT}$$

$$C_R = \frac{l}{A Y_R} = \frac{8.75 \cdot 39.4 \cdot 4}{\pi \left(\frac{3}{8}\right)^2 \; 10.3 \cdot 10^{10}} = .0304 \cdot 10^{-6} \frac{M}{NEWT}$$

$$C_S = \frac{C_R C_M'}{C_R + C_M'} = \frac{.0304 (.0060) \cdot 10^{-6}}{.0364} = .0050 \cdot 10^{-6} \frac{M}{NEWT}$$

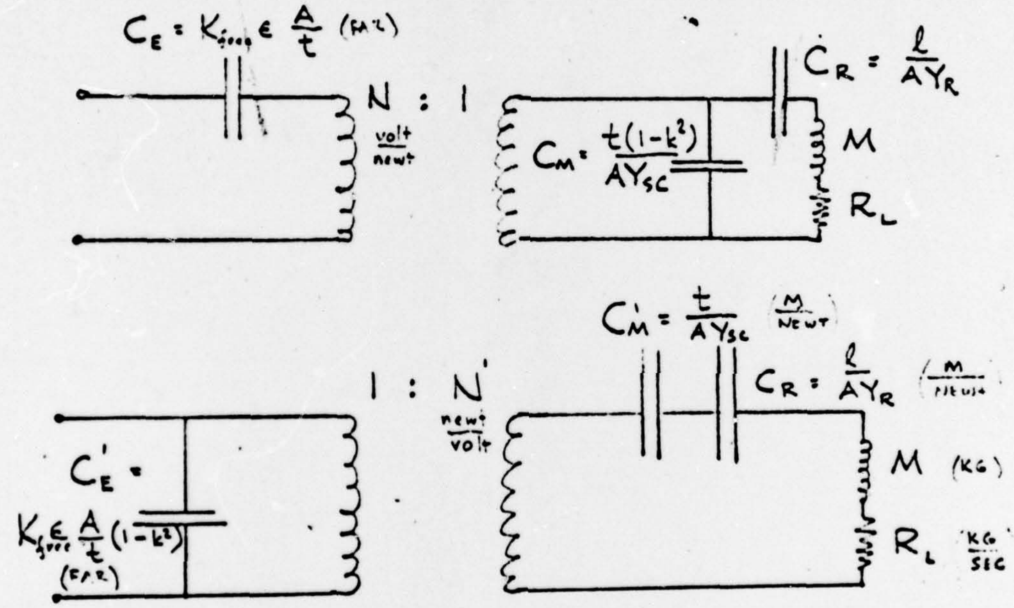
$$a = \frac{C_R}{C_M'} = \frac{.0304}{.0060} = 5.06$$

$$\frac{k'^2}{1 - k'^2} = \frac{a}{a+1} \left(\frac{k^2}{1 - k^2} \right) \qquad k' = .303$$

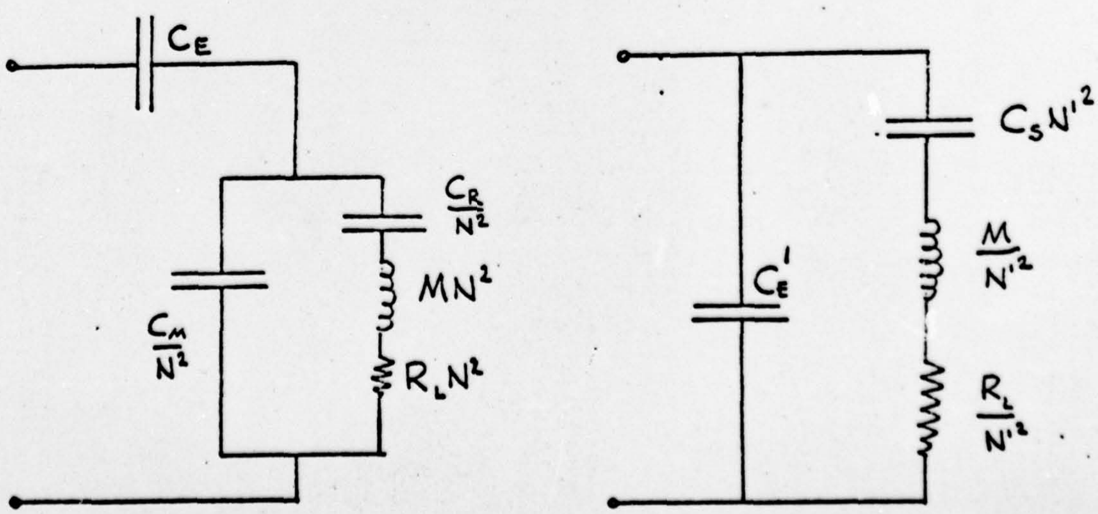
THIS VALUE OF k IS THEORETICAL. EXPERIENCE HAS SHOWN THAT ACTUAL VALUE IS LOWER. ONE MUST KNOW k TO SOLVE FOR C_E', C_M, N AND N'.

IF THE AIR IMPEDANCE LOOP OF THE SOURCE IS AVAILABLE ONE MAY SOLVE FOR THE ACTUAL k DIRECTLY FROM THE INDICATED ANTIRESONANCE (MAX IMPEDANCE POINT IF DAMPING IS SMALL). THIS WILL BE DONE BELOW.

THE TWO EQUIVALENT CIRCUITS OF THE TRANSDUCER IN AIR ARE GIVEN BELOW:



ELIMINATING THE TRANSFORMERS AND COMBINING C'_M AND C_R INTO C_S WE OBTAIN:



APPLICABLE RELATIONS FOR ABOVE CIRCUITS:

$$N'^2 = \frac{C_E - C_E'}{C_M'} \qquad N^2 = \frac{C_M' - C_M}{C_E}$$

$$N'^2 N^2 = k^4$$

BOTH CIRCUITS ARE IDENTICAL WHEN VIEWED FROM THE TERMINALS AND HAVE THE SAME RESONANCE AND ANTIRESONANCE:

RESONANCE (IF R_L IS SMALL) IS THE MINIMUM IMPEDANCE POINT. AT THAT FREQUENCY $C_S N'^2$ RESONATES WITH $\frac{M}{N'^2}$:

THUS: $\frac{1}{\omega_r C_S N'^2} = \omega_r \frac{M}{N'^2} \qquad \omega_r^2 = \frac{1}{C_S M}$

AND: $\omega_r = \frac{1}{\sqrt{C_S M}} = \sqrt{\frac{\frac{1}{C_M'} + \frac{1}{C_R}}{M}}$

NOTE: ω_r DOES NOT DEPEND ON k

ANTIRESONANCE (IF R_L IS SMALL) IS THE MAXIMUM IMPEDANCE POINT. AT THAT FREQUENCY $C_S N'^2$ AND $\frac{M}{N'^2}$ ARE IN PARALLEL RESONANCE WITH C_E':

THUS: $\frac{1}{\omega_{ar} C_E'} = \frac{\omega_{ar} M}{N'^2} - \frac{1}{\omega_{ar} C_S N'^2}$

AND: $\omega_{ar} = \sqrt{\frac{N'^2\left[\frac{1}{C_E'} + \frac{1}{C_S N'^2}\right]}{M}} = \sqrt{\frac{\frac{1}{C_M'(1-k^2)} + \frac{1}{C_R}}{M}}$

NOTE: ω_{ar} DEPENDS ON k

THUS FOR THIS SOURCE

RESONANCE

$$M = \frac{5.8}{2} = 2.9\ \text{KG}$$

$$\omega_r = \frac{1}{\sqrt{2.9 \quad .0050 \cdot 10^{-6}}} = 8300;\ f_r = 1320\ \text{cps}$$

ACTUAL = 1318 cps

ANTIRESONANCE

$$\omega_{ar} = \sqrt{\frac{\dfrac{10^6}{.0060(1-.303^2)} + \dfrac{10^6}{.0304}}{2.9}} = 8650;\ f_{ar} = 1380\ \text{cps}$$

ACTUAL = 1335 cps

VALUE OF k MUST BE LOWERED TO OBTAIN BETTER AGREEMENT.

$$\sqrt{\frac{\dfrac{10^6}{.0060(1-k^2)} + \dfrac{10^6}{.0304}}{2.9}} = 8400$$

1335 (6.28)

$$k = .152$$

CIRCUIT PARAMETERS:

$$C_E = .0745\ \mu F$$

$$C_E' = C_E(1-k^2) = .0726\ \mu F$$

$$C_M' = .0060 \cdot 10^{-6}\ \frac{M}{\text{NEWT}}$$

$$C_M = C_M'(1-k^2) = .0585 \cdot 10^{-6}\ \frac{M}{\text{NEWT}}$$

$$C_R = .0304 \cdot 10^{-6}\ \frac{M}{\text{NEWT}}$$

$$C_S = .0050 \cdot 10^{-6}\ \frac{M}{\text{NEWT}}$$

$$N'^2 = \frac{C_E - C_E'}{C_M'} = \frac{.0745 - .0726}{.0060} = .317\ \frac{\text{NEWT}^2}{\text{VOLT}^2}$$

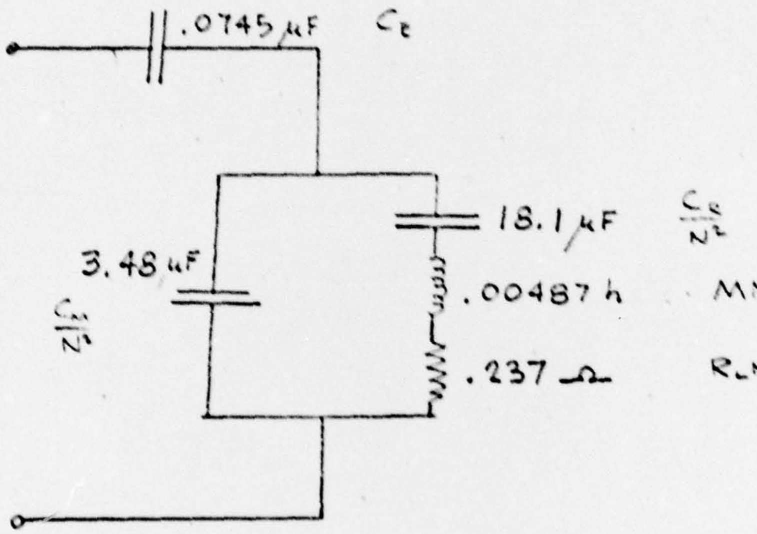
$$N^2 = \frac{k^4}{N'^2} = .00168\ \frac{\text{volt}^2}{\text{NEWT}^2}$$

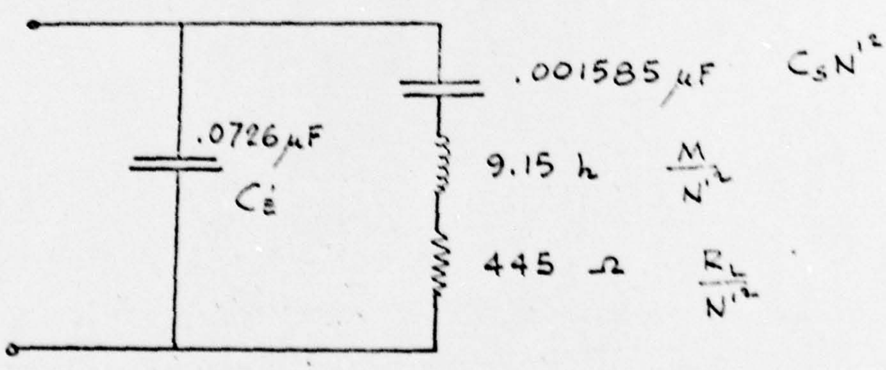
AT ANTIRESONANCE THE MEASURED AIR IMPEDANCE CIRCLE DIA = 5000 Ω. FROM THIS INFORMATION ONE CAN SOLVE FOR R_L. FOR THE FIRST CIRCUIT IT CAN BE SHOWN THAT AT ANTIRESONANCE THE RESISTIVE COMPONENT IS

$$\frac{Z_s^2 + (R_L N^2)^2}{R_L N^2} \approx \frac{Z_s^2}{R_L N^2} = 5000\,\Omega$$

$$Z_s = \omega(.00487) - \frac{10^6}{\omega \cdot 18.1} \qquad Z_{s@1335} = 34.44\,\Omega \quad \therefore R_L N^2 = .237\,\Omega$$

$$R_L = 141\ \frac{KG}{SEC}$$





10-9-. SL (6)
WT-2 1200
AIR

f / ω	Z_S	Z_{CM}	$\left(\frac{R}{R^2+Z_S^2}\right)$	$\frac{Z_S}{R^2+Z_S^2}$	$\left[\frac{1}{Z_{CM}} - \frac{Z_S}{R^2+Z_S^2}\right]$	$(\ )^2+[\ ]^2$	$R+jX$	Z_{CE}	X_{TOT}
1320 8293.6	33.73	34.65	.2083	29.640	-.780	.653	320 +j 1195	-j 1618	-j 423
1335 8387.8	34.25	34.26	.2020	29.197	-.011	.0418	4840 +j 530	-j 1600	-j 1070
1340 8419.2	34.44	34.13	$.1998 \cdot 10^{-3}$	$29.036 \cdot 10^{-3}$	$+.264 \cdot 10^{-3}$	$.1095 \cdot 10^{-6}$	1820 -j 2410	-j 1594	-j 4004
1360 8544.9	35.25	33.63	.1908	28.381	+1.354	1.86	102 -j 730	-j 1571	-j 2301

THESE POINTS AGREE WELL WITH MEASUREMENTS

WT-2 (1200~) IN WATER

λ @ 1200 = 4'

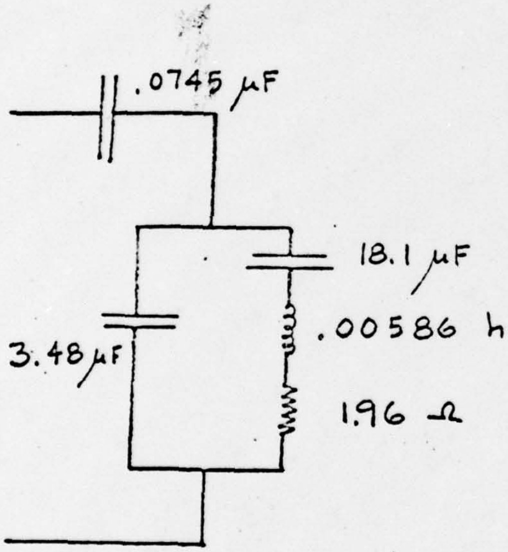
$$kr = \frac{2\pi}{4}\,\frac{3}{12} = .393 \rightsquigarrow Z_{AC} = \rho c A(.075 + j.325)$$

$$Z_{AC} = 1500 \cdot 1000\, \frac{\pi\,(3)^2}{4\,(39.4)^2}\,(.075 + j.325)$$

$$= 2050\,\frac{KG}{SEC} + j\,8890\,\frac{KG}{SEC}$$

@ 1200 cps: $\frac{8890}{2\pi(1200)} = 1.18$

EQUIVALENT CIRCUIT IN WATER



$$M = 2.9 + \frac{1.18}{2} = 3.49 \text{ KG}$$

$$R_L + R_{AC} = 141 + \frac{2050}{2} = 1166\,\frac{KG}{SEC}$$

ANTIRESONANCE:

$$\omega_{ar}(.00586) = \frac{10^6}{\omega_{ar}\,18.1} + \frac{10^6}{\omega_{ar}\,3.48}$$

$$\omega_{ar} = 7650 \qquad f_{ar} = 1220$$

CIRCLE DIA

$$Z_s = .00586\,(7650) - \frac{10^6}{7650\,(18.1)} = 37.7$$

$$\frac{Z_s^2}{R} = \frac{(37.7)^2}{1.96} = 725\ \Omega$$

RESONANCE

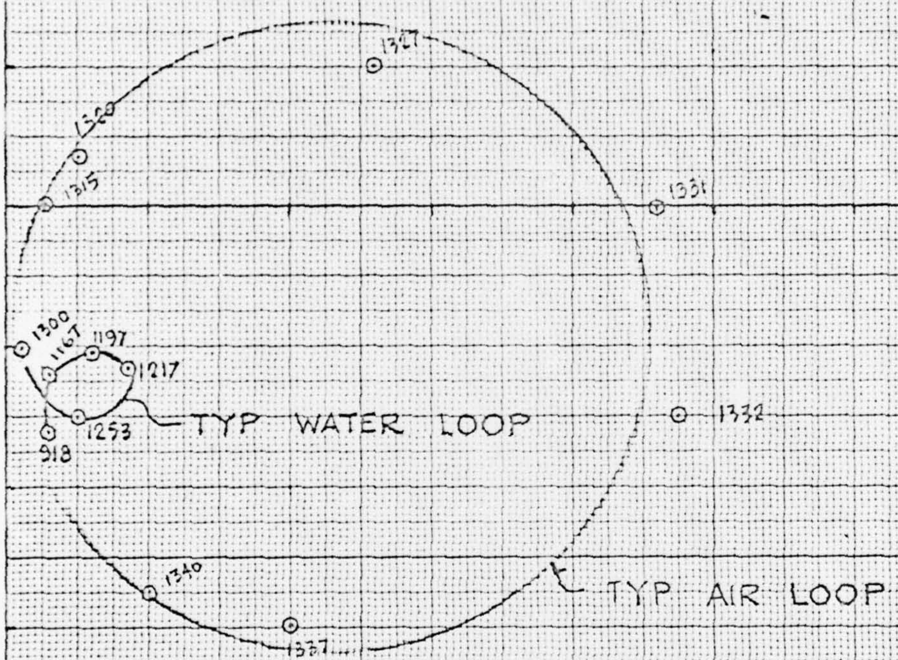
$$\frac{3.49}{.317} = 11.0\ h$$

$$\omega_r = \frac{1}{\sqrt{11.0\,(.001585)\,10^{6}}} \qquad f_r = 1205 \text{ cps}$$

THIS AGREES WELL WITH MEASUREMENTS

10-9-64 (8)
WT.2 (200)
WATER

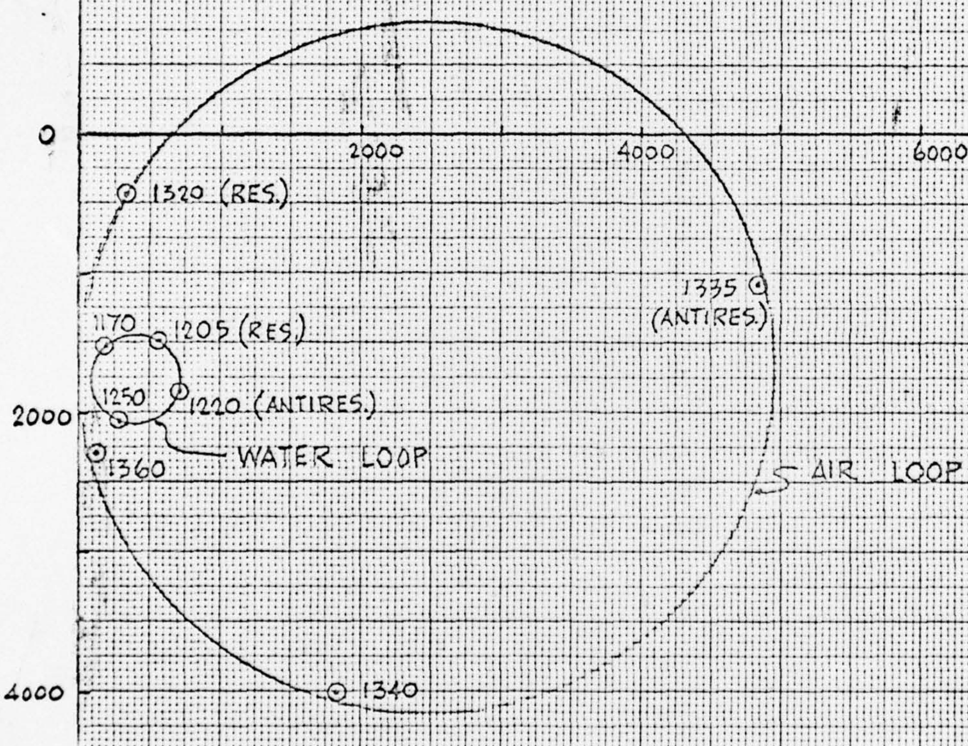
f / ω	Z_s	Z_{cm}	$\left(\frac{R}{R^2+Z_s^2}\right)$	$\frac{Z_s}{R^2-Z_s^2}$	$\left[\frac{1}{Z_{cm}}-\frac{Z_s}{R^2+Z_s^2}\right]$	$()^2+[]^2$	$R+jX$	Z_{CE}	X_{TOT}
1170							183		
7351.1	35.56	39.09	$1.545 \cdot 10^{-3}$	$28.044 \cdot 10^{-3}$	$-2.463 \cdot 10^{-3}$	$8.44 \cdot 10^{-6}$	+j 293	-j 1820	-j 1527
1205							585		
7571.0	37.07	37.95	1.422	26.901	-.651	2.435	+j 268	-j 1770	-j 1502
1220							714		
7665.3	37.71	37.49	1.374	26.445	+.229	1.93	-j 119	-j 1750	-j 1869
1250							269		
7853.8	38.99	36.59	1.284	25.550	+1.78	4.79	-j 372	-j 1710	-j 2082



TYP MEASURED LOOPS

WT-2 (1200)

SL 10-12-64



COMPUTED LOOPS

WT-2 (1200)

SL 10-12-64